Seat No. : _______________

DA-114

December-2018

M.Sc., Sem.-I

401 : Mathematics
(New)

Time : 2:30 Hours] **[Max. Marks : 70**

Instruction : **All** questions are compulsory. Normal Probability distribution tables are provided. Use of non-programmable scientific calculator is allowed.

1. (A) Attempt the following : **14**

(i) State (only) the Bayes' theorem.

A laboratory blood test is 95 percent effective in detecting a certain disease, when it is, in fact, present. However, the test also yields a 'false positive' result for 1 percent of the healthy persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he or she has the disease). If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that the test result is positive ?

(ii) State and prove the Chebyshev's inequality. State its usefulness.

In the computing industry the average age of professional employees tends to be younger than in many other business professions. Suppose the average age of a professional employed by a particular computer firm is 28 with a standard deviation of 6 years. Determine within what range of ages would at least 80% of the workers ages fall.

OR

(i) Let $f(x) = ke^{-ax}(1 - e^{-ax})$, x, $a > 0$.

Find k such that $f(x)$ is a density function. Find the corresponding cumulative distribution function. Find $P(x > 1)$.

(ii) Consider the following probability density function :

$$f_x(x) = 4x \qquad 0 \le x \le ½$$
$$= 4(1 - x) \qquad ½ \le x \le 1$$
$$= 0 \qquad \text{otherwise}$$

Show that it is indeed a pdf. Obtain the distribution function $F_x(x)$.

(B) Answer very briefly any **four** : **4**

(i) A system composed of n separate components is said to be a parallel system if it functions when at least one of the component functions. For such a system, if component i, independent of other components, functions with probability p_i, i = 1, 2, 3, ... n, what is the probability that the system functions ?

(ii) The probability that the consumer will be exposed to an advertisement of a certain product by seeing a commercial on television is 0.04. The probability that the consumer will be exposed to the product by seeing an advertisement on a billboard is 0.06. The two events being assumed to be independent, what is the probability that the consumer will be exposed to both advertisements ?

(iii) The probability that the consumer will be exposed to an advertisement of a certain product by seeing a commercial on television is 0.04. The probability that the consumer will be exposed to the product by seeing an advertisement on a billboard is 0.06. The two events being assumed to be independent, what is the probability that the consumer will be exposed to at least one of the advertisements ?

(iv) An infinite sequence of independent trials is to be performed. Each trial results in a success with probability p and a failure with probability 1-p. What is the probability that at least 1 success occurs in the first n trials ?

(v) Roughly sketch a Box Plot and label it.

(vi) Roughly sketch a positively and negatively skewed distribution curve.

2. (A) Attempt the following : **14**

(i) The continuous random variable W has the PDF $f_w(w) = 12w^2(1 - w), 0 < w < 1$.

Calculate $P(w) < \frac{1}{2}$ and determine an expression for the CDF $F_w(w)$.

Also calculate E(W), Var(W).

(ii) A continuous random variable has a probability density function $f_x(x) = ke^{-2x}, x > 0$.

Find k and $P(X < 5.27)$. Also calculate E(X) and Var(X). Find the median and the interquartile range.

OR

(i) If X is a continuous random variable with probability density function f_x that satisfies $f_x(x) > 0$ for $a < x < b$, and if $y = H(x)$ is a continuous strictly increasing or strictly decreasing function of x, then show that the random variable $Y = H(X)$ has density function $f_y(y) = f_x(x) \cdot \left|\frac{dx}{dy}\right|$

with $x = H^{-1}(y)$ expressed in terms of y.

Suppose that the random variable X has the following density function :

$$f_x(x) = \frac{x}{8} \qquad 0 \le x \le 4$$
$$= 0 \qquad \text{otherwise}$$

For the random variable $Y = H(X)$ where $H(x) = 2x + 8$, find the probability density function f_y of Y.

(ii) The probability density function of the random variables $[X_1, X_2]$ is given by

$$f(x_1, x_2) = \frac{k}{1000}, 0 \le x_1 \le 100, 0 \le x_2 \le 10$$
$$= 0, \text{ otherwise}$$

Find the appropriate value of k, marginal densities of X_1 and X_2 and the expression for the cumulative distribution function $F(x_1, x_2)$.

(B) Answer very briefly any **four** : **4**

(i) Consider the probability density function $f_x(x) = k.\sin x,\ 0 \le x \le \frac{\pi}{2}$. Find the appropriate value of k. Find the mean of the distribution.

(ii) Write the probability mass function of the Pascal distributed random variable X. Write its mean and SD.

(iii) State the pdf of a hypergeometric distribution. State its mean and SD.

(iv) State the pdf of an exponential distribution. State its mean and SD.

(v) The joint density of $[X_1, X_2]$ is given by

$$f(x_1, x_2) = 6x_1 \qquad 0 < x_1 < x_2 < 1$$
$$= 0 \qquad \text{otherwise}$$

Find the marginal of X_1.

(vi) The joint density of $[X_1, X_2]$ is given by

$$f(x_1, x_2) = 6x_1 \qquad 0 < x_1 < x_2 < 1$$
$$= 0 \qquad \text{otherwise}$$

Find the marginal X_2.

3. (A) Attempt the following : **14**

(i) State the pdf of a Poisson distribution. State its application. In standard notation, develop the Poisson distribution as a limiting form of the Binomial distribution.

(ii) For a random variable that follows the binomial distribution, find the first and the second moments about the origin and the second central moment. For a random variable that follows the gamma distribution, find the first and the second moments about the origin.

OR

(i) Determine the moment generating function of the two parameter random variable X, defined by the probability density function $f(x) = \lambda e^{-\lambda(x - a)}$, $x \geq \alpha$; $\lambda, \alpha > 0$. Determine the mean and variance of X.

(ii) Write the pdf of the Weibull distribution . State its various parameters. State its one important area of application.

The diameter of steel shafts is Weibull distributed with parameters $\gamma = 1.0$ inches, $\beta = 2$, and $\delta = 0.5$. Find the probability that a randomly selected shaft will not exceed 1.5 inches in diameter.

(B) Answer very briefly any **three** : **3**

(i) Give the condition under which the DeMoivre-Laplace Approximation is fairly good.

(ii) An electric component is known to have a useful life represented by an exponential density with mean failure rate of 10^{-5} failures per hour. Find the percentage of such components that would fail before the mean life.

(iii) Which are the distributions having the memoryless property ?

(iv) Define the moment-generating function of the probability distribution of a random variable X.

(v) What is the principal advantage of using the characteristic function over a moment generating function ?

4. (A) Attempt the following : **14**

(i) State the pdf of the Beta distribution. Show that when both the shape parameters of the Beta distribution are 1, the Beta distribution reduces to the Uniform distribution. Graph the density function. Also show that when the parameters are (2, 1) or (1, 2), the Beta distribution reduces to the triangular probability distribution. Graph the density function.

(ii) The probability that an item produced by a certain machine will be defective is 0.01. Find the probability that the random sample of 100 items selected at random from the total output will contain no more than one defective item.

A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused. Use $e^{-1.5} = 0.2231$.

OR

(i) Explain the reproductive property of the Normal Distribution.

An assembly consists of three linkage components X_1, X_2 and X_3 in series. Let $Y = X_1 + X_2 + X_3$. The properties of X_1, X_2 and X_3, are given below, with means in centimetres and variance in square centimetres.

$X_1 \sim N(12, 0.02)$

$X_2 \sim N(24, 0.03)$

$X_3 \sim N(18, 0.04)$

If X_1, X_2 and X_3 are independent, determine $P(53.8 \leq Y \leq 54.2)$

(ii) X_1, X_2, X_3 and X_4 are independent random variables. Let $Y_1 = \ln X_1 \sim N(4, 1)$, $Y_2 = \ln X_2 \sim N(3, 0.5)$, $Y_3 = \ln X_3 \sim N(2, 0.4)$, $Y_4 = \ln X_4 \sim N(1, 0.01)$.

For $W = e^{1.5} [X_1^{2.5} X_2^{0.2} X_3^{0.7} X_4^{3.1}]$, find $P(20{,}000 \leq W \leq 600{,}000)$

(B) Answer very briefly any **three** : **3**

(i) State the Central Limit Theorem along with the necessary general conditions.

(ii) 250 small parts are packaged in a crate. Part weights are independent random variables with a mean of 0.5 pound and a standard deviation of 0.10 pound. Twenty crates are loaded to a pallet. Find the mean and standard deviation of the total weight of the pallet.

(iii) The incomes of a group of 10,000 persons were found to be normally distributed with mean ₹ 750 and standard deviation ₹ 50. Approximately what percent of the group had income exceeding ₹ 668 ?

(iv) The incomes of a group of 10,000 persons were found to be normally distributed with mean ₹ 750 and standard deviation ₹ 50. Approximately what percent of the group had income exceeding ₹ 832 ?

(v) The incomes of a group of 10,000 persons were found to be normally distributed with mean ₹ 750 and standard deviation ₹ 50. What was the lowest income among the richest 100 ?